There are two levels of identification in Relational Data Problems:

1) identifying the objective function for purposes of finding the optima;

2) identifying the objective function for purposes of exploring it with Markov Chain Monte Carlo Methods.

For (1), the number of constraints is equal to \( s(s+1)/2 \). Namely, one point is placed at the origin for \( s \) constraints, a second point has \( s-1 \) coordinates placed at 0.0, a third point has \( s-2 \) coordinates placed at 0.0, etc. This fixes the rotation.

To see this consider \( s=2 \). Fix one point at the origin and then the rotation is fixed by selecting \( \theta \):

\[
\Gamma = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad 0 \leq \theta \leq 2\pi
\]

This is equivalent to placing one coordinate of a second point at 0.0

For (2), we need \( s(s+2)/2 + 2^s - 1 \) constraints. To see this, given a specific \( \theta \) we have four rotation matrices:

\[
\Gamma_1 = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} -\cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad \Gamma_3 = \begin{bmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & -\cos \theta \end{bmatrix} \quad \Gamma_4 = \begin{bmatrix} -\cos \theta & -\sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}
\]

Or
\[ \Gamma^* = \Delta \Gamma \text{ where } \Delta = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix} \] (14)

That is, given a specific \( \theta \), there are \( 2^s \) sign flips corresponding to the \( s \) columns of the rotation matrix. In practice we have placed \( s(s+1)/2 \) zeroes in the coordinate matrix and this arbitrarily selections one rotation matrix. Hence, there are \( 2^s - 1 \) further possible rotation matrices corresponding to the remaining sign flips.