The One Parameter logistic model (the Rasch model)

Let $\theta_j$, $j=1,\ldots,q$, be the jth subject's level of ability.

Let $\delta_i$, $i=1,\ldots,p$, be the ith item -- the level of difficulty of the ith question.

Then the probability that subject $i$ gets test question $j$ correct is:

$$P(z_{ij} = 1 | \theta_j, \delta_i) = \frac{e^{\alpha(\theta_j - \delta_i)}}{1 + e^{\alpha(\theta_j - \delta_i)}}$$

where $z_{ij}$ is an indicator variable that is equal to 1 if subject $j$ answers question $i$ correctly, and 0 otherwise; and $\alpha$ determines the shape of the item response function. If $\alpha \to \infty$ then $P(z_{ij} = 1)$ if $\theta_j > \delta_i$. That is, it collapses to a Guttman Scale.

Assuming independence of the responses; that is:

$$P(z_{ij} = 1 \cap z_{ij} = 0) = P(z_{ij} = 1)P(z_{ij} = 0)$$

Then

$$\frac{P(z_{ij} = 1 \cap z_{ij} = 0)}{P(z_{ij} = 0 \cap z_{ij} = 1)} = \frac{P(z_{ij} = 1)P(z_{ij} = 0)}{P(z_{ij} = 0)P(z_{ij} = 1)} =$$

$$\frac{e^{\alpha(\theta_j - \delta_i)} * 1}{1 + e^{\alpha(\theta_j - \delta_i)}} \cdot \frac{1}{1 + e^{\alpha(\theta_j - \delta_i)}} = \frac{e^{\alpha(\theta_j - \delta_i)}}{e^{\alpha(\theta_j - \delta_i)}} = e^{\alpha(\delta_j - \delta_i)}$$

So the individual parameter cancels leaving just the difference between the two question parameters.