The Workshop

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*Dimensionland: An Excursion into Spaces**

Scaling analysis is based on a geometric metaphor. This workshop paper examines how our understanding of the metaphor affects our use of scaling. Instances which appear to be multidimensional are shown to be unidimensional under other scaling models. Conversely, some apparently unidimensional cases are found to be better described as multidimensional. Particular attention is given to the difference between multidimensional scaling and factor analysis. The philosophical implication of our dependence on the definition of unidimensionality is that scaling seeks only partial images of a real world that may be fundamentally unknowable.

Scaling analysis seeks the latent dimensions underlying a set of obtained observations. The variation across a set of variables is explained in terms of the different locations of these variables on hypothesized underlying dimensions. The dimensions are presumed to exist since their presence can make the variation across the variables explicable. Scaling techniques are used for two distinct purposes: description of data structure and measurement of individual behavior. The goal may be to describe the dimensionality of a set of variables—as in determining the dimensions underlying a party system. Or the intention may be to derive unidimensional indices on which individuals can be scored—as in constructing a scale of political efficacy which can be correlated with other attitudinal and behavioral measures.[1]

*This article benefits from Clyde Coombs's ideas on scaling models, from my collaboration with Richard Niemi and Jerrold Rusk on related projects, from the challenging comments of Lutz Erbring, Robert Friedrich, George Rabinowitz, and Stuart Thorson, from the suggestions by John Champlin and especially Sally Friedman for my leisure reading, and from the inspiration provided by a martyred square.

[1] The terms dimensional analysis and scaling will be used interchangeably in this paper, with factor analysis being included in this general rubric. The important general works on this subject include: Lee Anderson, Meredith Watts, and Allen Wilcox, *Legislative Roll-Call Analysis* (Evanston: Northwestern University Press, 1966); Clyde Coombs, *A Theory of Data* (New York: John Wiley and Sons, 1964); Harry Harman, *Modern Factor Analysis*, 2nd ed. (Chicago: University of Chicago Press, 1967); Duncan

But what is unidimensionality? The term is so familiar that we are easily lulled into fallacies in its usage. We assume it has a single meaning, so the dimensions produced by different scaling techniques are equivalent. Yet unless a formal proof is provided, there is no reason to believe that two scaling techniques have similar conceptions of what constitutes unidimensionality. Instead, unidimensionality may have different meanings which are appropriate in different substantive situations. As a result, a scaling technique may report that two dimensions underlie a set of data, even though the data might be considered unidimensional under some other conception of unidimensionality. Conversely, a unidimensional result may be obtained, although a two-dimensional representation would better satisfy the analyst's purposes. Thus a limited view of dimensions can restrict our success in determining dimensionality and in measuring individual positions.

A spatial analogy is intrinsic to scaling. A geometric model is used to represent certain of the relations among variables, with selected features of a geometric space being used to represent specific features of the observed data. Yet whenever we employ an analogy, our understanding of the phenomenon becomes limited by our understanding of the analogue. Spatial reasoning would not be helpful in communicating with a culture which does not employ geometric concepts. Similarly, the usefulness of dimensional analysis is limited by our own inabilities to comprehend fully the basics of geometry. Our restricted understanding of geometry limits our interpretation of the term "unidimensionality" and hinders our use of scaling.

How our limited understanding of the geometric analogue and how our limited use of the term unidimensionality restrict our ability to use scaling in studying political phenomena are but special cases of how our use of language can circumscribe our conception of the world.[2] The theme of how language limits and is limited by perceptions of the world is very general. "We learn language and learn the world *together,* . . . they become elaborated and distorted together and in the same places."[3] Terms have meaning only to the

MacRae Jr., *Issues and Parties in Legislative Voting* (New York: Harper and Row, 1970); R. Rummel, *Applied Factor Analysis* (Evanston: Northwestern University Press, 1970); Herbert Weisberg, "Dimensional Analysis of Legislative Roll Calls," (unpublished Ph.D. dissertation, The University of Michigan, 1968). Measurement concerns are not of central importance here, but they are further discussed in Brian Ellis, *Basic Concepts of Measurement* (Cambridge: Cambridge University Press, 1966) and Warren Torgerson, *Theory and Methods of Scaling* (New York: John Wiley and Sons, 1964).

[2] See, for example, Hanna Pitkin's discussion of the themes discussed in this paragraph in *Wittgenstein and Justice* (Berkeley: University of California Press, 1972).

[3] Stanley Cavell, *Must We Mean What We Say?* (New York: Charles Scribner's Sons, 1968), p. 19.

extent to which our experiences supply meaning, so our experiential base inevitably restricts our language use. Conversely our understanding of phenomena is necessarily limited by our bounded store of concepts and terms. "The concepts we have settle for us the form of the experience we have of the world."[4] In the extreme, this may mean that "real" world phenomena seem to exist only insofar as we have developed terms with which to describe them. Thus we may be unable to recognize some phenomena as unidimensional until we recognize the full scope of the term.

When unidimensionality does not seem to suffice for a given set of data, we resort to more complicated multidimensional explanations. More generally stated, when events occur without a simple explanation, we devise complex causal mechanisms to justify their occurrence. However the apparent complexity may be due merely to the inadequacy of our concepts. Extension of our concepts may permit the ready comprehension of seemingly complex events. Simplicity always exists only with reference to a body of theory—events are never simple in themselves but only as part of a familiar framework. Yet Abraham Kaplan's "paradox of conceptualization" intrudes here: "the proper concepts are needed to formulate a good theory, but we need a good theory to arrive at the proper concepts."[5] Apparent multidimensionality may be due to an insufficiently general conception of unidimensionality, in which case much seemingly complex behavior would appear unidimensional if we revise our understanding of the term. Still our experience—with the geometry of the real world and with the common scaling approaches—makes it difficult for us to broaden sufficiently our conception of unidimensionality.

The role of spatial analogy in our comprehension of the real world is best made by reference to Edwin Abbott's classic tale of *Flatland.*[6] Flatland is a world of two dimensions whose inhabitants are triangles, squares, and the like. The women are lines, the men are shapes with angles; the greater the number of angles a person has, the higher is his class. The residents of this society cannot see outside of their plane, so all they can see are the line segments in their plane. They cannot conceptualize the existence of a third dimension since they are incapable of perceiving it. *Flatland* relates the saga of how A Square came to be the first citizen of his world to realize that a third dimension exists.

[4] Peter Winch, *The Idea of a Social Science and Its Relation to Philosophy,* ed. by R. F. Holland (New York: Humanities Press, 1965), p. 15.

[5] Abraham Kaplan, *The Conduct of Inquiry* (San Francisco: Chandler Publishing, 1964), p. 53.

[6] Edwin Abbott, *Flatland,* 6th ed. (New York: Dover Publications, 1952).

Events occur which defy understanding within Flatland's limited conception of space. For example, a sphere tries proving to the square that a third dimension exists by moving up off the Flatland plane. At first the sphere is a large circle (for all the square can see is the sphere's intersection with Flatland's plane), then a smaller circle as the sphere rises, then a dot, and finally the sphere disappears. Yet the square cannot comprehend this event since it is beyond accepted theories, and he puts it down as magic.

Another incident is the square's magical mystery tour to Lineland. Lineland is a world of one dimension whose inhabitants are line segments arrayed along the dimension. These line segments have length but no width, have fixed positions on the line, cannot move outside the line, and cannot understand how an outsider–the square–can "see" their order on the line. The square assumes his vision of Lineland is a bad dream, rather than realizing that careful examination of the difference between Lineland and Flatland would suggest the existence and nature of a three-dimensional world.

Finally the sphere bumps the square off the Flatland plane so the square can look down and see the structure of Flatland. He can see the insides of objects for the first time, since on the Flatland plane only object edges are visible. Suddenly he realizes that there is a third dimension and even a fourth (which the sphere regards as so inconceivable that he departs angrily when the square presses the point). Unfortunately for the square the other Flatland residents do not believe his new insight into the nature of the world, and the square suffers the imprisonment with which societies protect themselves from original thinkers.

Sphereland, a sequel to *Flatland,* is authored by the square's grandson–A Hexagon.[7] Flatland has come to realize that there are three dimensions, though the implications are poorly understood. It evokes the state of science in Europe immediately following Columbus's discovery of the New World but prior to complete revision of scientific theories to account for the new findings. *Sphereland* relates a series of mysterious events–mysterious until the hexagon develops the proper geometric understanding. For example, triangles are measured with new more accurate calibrating devices and it is found that their angles sum to more than 180°. This absurd event becomes explicable only when it is realized that Flatland's plane is actually curved.

These fables are intended to help readers understand the nature of spaces of more than three dimensions. We can comprehend such spaces only by analogy, and reading about Flatland and Lineland can sharpen our powers of

[7] Dionys Burger, *Sphereland* (New York: Thomas Crowell, 1965), tr. Cornelie Rheinboldt.

analogy.[8] But *Flatland* also serves to emphasize how our inability to comprehend fully the basics of geometry limits our understanding of scaling. Like the square, we must be lifted out of our Flatland if we are to perceive the variety of dimensional forms. And we must extend our concepts to fit the complex world with a simple theory, rather than devising ad hoc explanations of why behavior does not always satisfy preconceived notions of unidimensionality.

The purpose of this paper is to explore the nature of unidimensionality. It rests on the assumption that most people are limited in their perceptions of dimensions and do not sufficiently question the use of the term. Six case studies will illustrate the variety of possible uses of the unidimensionality concept. In each instance what is multidimensional in one sense may be unidimensional in another, so our understanding of the world is directly affected by our interpretation of unidimensionality. What follows might thus be viewed as a series of mind-expanding games designed to explore the proper limits of the concept of unidimensionality.

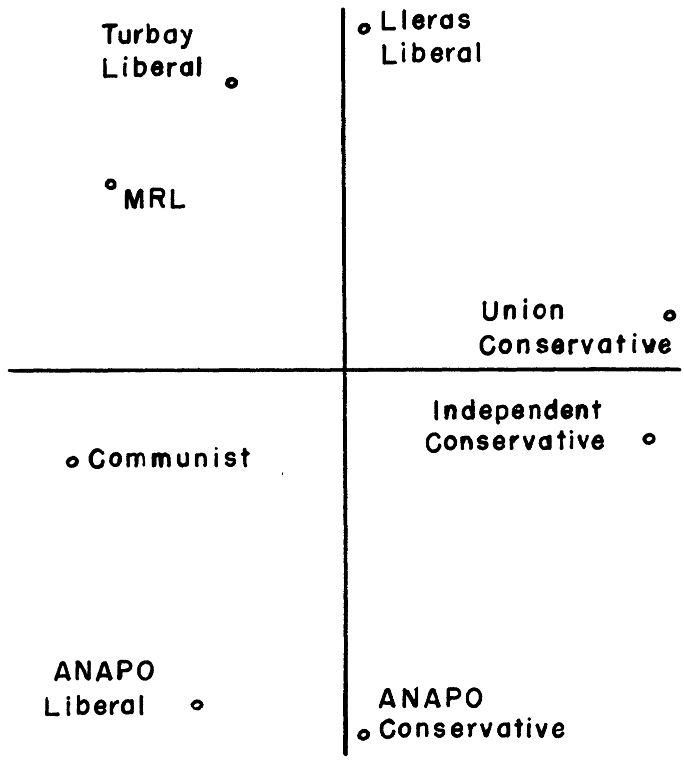
Circleland: An Empty World without Ends

A circle is a two-dimensional geometric shape. What could be more obvious—or less true? A circle is only a straight line whose two ends have been joined together. If only the circumference of the circle is considered (and not its interior), then the circle is certainly unidimensional. Most scaling studies, however, unquestioningly treat circular solutions as two-dimensional rather than realizing that they are essentially unidimensional.

When we obtain two-dimensional solutions, we generally accept the fact that there are two dimensions and we go on to look at the ordering of points on them to help name the dimensions. Too often we do not even bother to plot the points in order to examine their shape. The argument is that some multidimensional solutions have shapes that *can* be interpreted as unidimensional. If, for example, the solution is exactly circular, then there is a distinct sense in which all that matters is the relative position of the points along their circle rather than their projections on artificial axes. Two dimensions would be required if some points were within the circle, but if all the

[8] For example, the residents of Lineland cannot see each other's interiors, while the square looking down on Lineland can see their middle points. The residents of Flatland cannot see each other's interiors, but when the square is lifted above Flatland he can see into their insides. The thought-provoking conclusion—which *Flatland* wisely leaves unstated—is that a four-dimensional creature could see our intestines!

FIGURE 1
Hoskin and Swanson's Multidimensional Scaling of the Colombian Party System*



*From Gary Hoskin and Gerald Swanson, "Inter-Party Competition in Colombia," *American Journal of Political Science,* 17 (May 1973), 333.

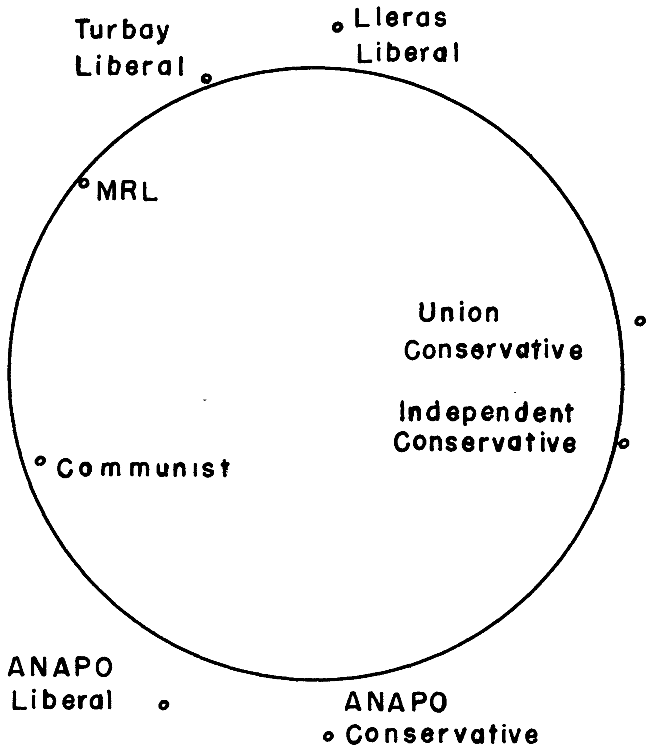
points fall along the circle then each point can be described in terms of a single parameter—the angle formed by the line connecting it with the origin and the 0° line.[9]

Circular shapes may be expected for alliance structures and for vote coalitions where extremists of the left and right coalesce for particular purposes.[10] An example is provided by the Hoskin and Swanson study of the

[9] If this is viewed in terms of polar coordinates, the radius is constant since all points are along the circumference of the circle, and the single varying parameter is the angle. A circular representation may be employed when the radius values are virtually equal.

[10] See also the discussion of the similarity of theories of the radical right and left in Ole Holsti, "The Study of International Politics Makes Strange Bedfellows," *American Political Science Review,* 68 (March 1974), 217–242.

FIGURE 2
Circular Scale of Colombian Party System

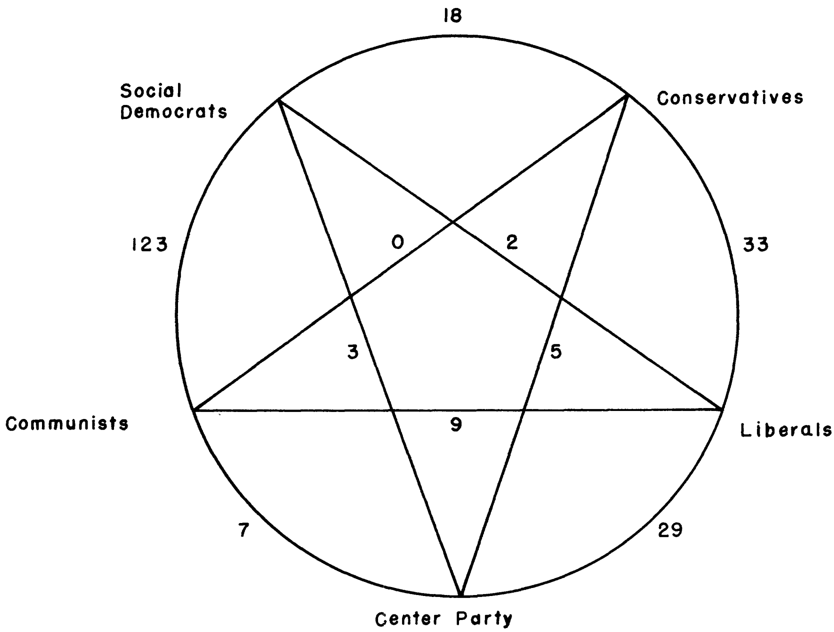


Colombian party system.[11] They asked leaders of Colombian political parties to rank order their preference for the several parties. Figure 1 reproduces their multidimensional scaling solution. They interpret this solution as involving a left-right dimension (the horizontal axis) and a government support-opposition dimension (the vertical dimension). However, note that there is a dependency between the dimensions such that no party is accorded a centrist position on both dimensions. Thus there is a sense in which this solution is essentially unidimensional, with party leaders ordering other parties by the distances from their own party along the circumference of the circle shown in Figure 2. The circle provides a very good fit to the solution.[12] Treating the

[11] Gary Hoskin and Gerald Swanson, "Inter-Party Competition in Colombia," *American Journal of Political Science*, 17 (May 1973), 316–350.

[12] A scaling algorithm could be developed to place parties closer together along the

FIGURE 3
Circular Scale of Swedish Party Voting in 1964 Riksdag*



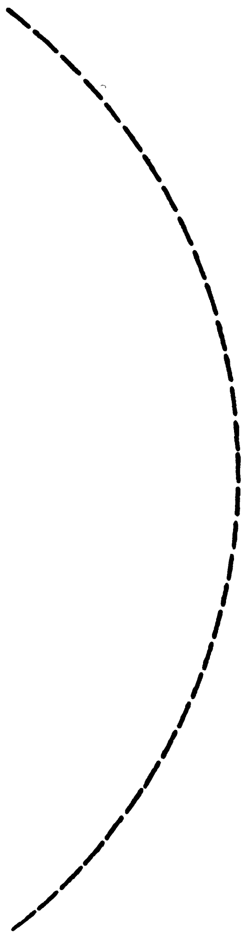
*The figures show the number of times each pair of parties voted together against the other three parties. The votes in the middle of the star between nonadjacent parties represent deviations from the circular scale. The data are from Nils Stjernquist and Bo Bjurulf, "Party Cohesion and Party Cooperation in the Swedish Parliament in 1964 and 1966," *Scandinavian Political Studies,* 5 (1970), Table 21, p. 151.

centrist Lleras Liberal and ANAPO Conservative parties as similar to one another on a left-right dimension would be to ignore their placement on opposite sides of the circle.

A circular pattern also fits voting coalitions in the Swedish Riksdag. Party cohesion is not perfect in the Riksdag, but it is so high that the parties can be considered the basic actors. The conventional unidimensional party order from left to right is: Communists, Social Democrats, Center party, Liberals, and Conservatives. However the best fitting dimensional pattern for their voting in 1964 is a circle with the Social Democrats moved between the

circle the more often they vote together, but this is more complex than it may appear, so adjacent parties have been equally spaced in Figure 2.

FIGURE 4
Hypothetical Curved Spatial Solution



Communists and Conservatives (see Figure 3).[13] Only on 19 of the 366 roll calls did nonadjacent parties in Figure 3 vote together, while 41 of the 366 roll calls would not fit if the Social Democrats and Communists were reversed. The ordering of the Social Democrats and Communists does not fit with the usual view of Swedish politics because the Social Democrats voted

[13] Data are from Nils Stjernquist and Bo Bjurulf, "Party Cohesion and Party Cooperation in the Swedish Parliament in 1964 and 1966," *Scandinavian Political Studies,* 5

more often with only the Conservatives than with only the Center party. And that is essentially a circular phenomenon–the parties of the left and the right sometimes vote together. The parties of the left and right are similar, at least in uniting against the center. A circular representation of this voting is appropriate to emphasize the closeness of the Social Democrats and Conservatives. Yet the data can be seen as unidimensional–but with the left and right extremes drawn together.[14]

When a computer program produces a circular solution, it is worth considering the unidimensional conceptualization of the circle. Even a curved solution should be examined as unidimensional. Figure 4 shows a hypothetical spatial solution. It is nothing more than a straight line that has been curved. Should this be interpreted as two-dimensional or unidimensional? Should the slight second dimension be given a substantive interpretation, or should we just examine the relations among the points themselves along the curved line? A solution of this shape should be considered suspect since the curvature might result from some distortion by the scaling technique rather than from the observed behavior. The points bear a unidimensional relationship to one another, and that essential unidimensionality should not be overlooked.[15]

The circular and curved cases directly raise the question of what is meant by a dimension. Perhaps the most basic definition would be to view a dimension as a one-parameter system where there are order relations on the parameter. Such a definition avoids the restriction of linearity. Yet it imposes

(1970), Table 21, p. 151. Similar results have been obtained for their 1966 data and from later data collected by Aage Clausen and Soren Holmquist.

[14] Another possible representation of the Swedish data would be as a unidimensional proximity scale, as described in the next section. A vote would fit that scale if only adjacent parties voted for it. But what if the three left parties and the right party vote for a bill while the remaining party votes against it? That would violate the proximity scale, since nonadjacent parties are voting together in support of a motion. A useful distinction here involves whether the agreement of nonadjacent parties is in opposition to proposals of the center (where the left and right vote together for opposite reasons) or in support of a common proposal to alter the status quo (as when the radical left and the conservative right unite to pass a bill providing for local control rather than the federal programs which the old left favors). The Swedish data are not available in a form to check whether the coalitions of left and right against the center are in opposition or in agreement to alter the status quo.

[15] Calculate the interpoint distances in Figure 4 using the Euclidean distance formula for two-dimensional space and then using the distances between the points along the curvature. Those two sets of distance figures will be monotone with one another. The unidimensional curvature representation loses no ordinal distance information that was present in a Euclidean two-space solution.

such few restrictions that it might allow a set of points to be considered unidimensional if they were in the shape of a numeral "2," "3," "5," "6," or even "8," where the one parameter would be how far from the beginning of drawing the numeral is each specific point. Are such cases "unidimensional?" How far is it *useful* to push our geometric conception of a dimension? For some purposes even these numerals might be regarded as unidimensional, though for most purposes an explanation of the shape would be required which would entail a two-dimensional view.

This discussion of circular dimensions is not intended to demonstrate that the Colombian or Swedish cases are circular. Substantive experts may or may not accept the circular representation. The purpose at present is to indicate that there is a reasonable model which is rarely used but which can be appropriate. It would be of interest to know whether the model fits these two polities, but it suffices to suggest the possible relevance. The shape cannot be tested unless the structure is first hypothesized as done there. Hopefully the hypothesis alone produces some insights which are substantively interesting.

Proximityland: A Conflictual World without Comparisons

Guttman scaling, one of the first scaling techniques, tests a cumulative view of the world: attitudes are unidimensional only if everyone willing to accept one statement is willing to accept all easier statements. An alternative view of a dimension is the proximity notion: attitudes are unidimensional if everyone is acceptant only of adjacent statements. For example, in the cumulative case, a legislator would be willing to support an appropriation value up to a certain amount; he might support any amount up to $3 billion while opposing any greater expenditure. In the proximity case the legislator might instead feel the program would be worthwhile only if it received reasonable funding (say $2–4 billion), but would oppose appropriations of less as worthless and would oppose appropriations of more as wasteful; only adjacent amounts within the reasonable range would be supported. The order imposed by adjacencies is as well defined as the order of cumulation.[16]

Alker has suggested that the proximity notion is appropriate for voter approval of political candidates.[17] Table 1 shows his hypothetical scale for

[16] The differences between the two models are presented in Herbert Weisberg, "Scaling Models for Legislative Roll-Call Analysis," *American Political Science Review,* 66 (December 1972), 1306–1315.

[17] Hayward Alker, Jr., "Statistics and Politics," in Seymour Martin Lipset, ed., *Politics and the Social Sciences* (New York: Oxford University Press, 1969), Table 2, p. 276.

TABLE 1

A Hypothetical Proximity Scale of 1964 Presidential Contenders*

State	*Candidate* Ful-bright	Kennedy	Rocke-feller	Romney	Humphrey	Johnson	Jackson	Nixon	Reagan	Goldwater
Massachusetts	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No
New York	No	Yes	Yes	Yes	Yes	Yes	No	No	No	No
Michigan	No	No	Yes	Yes	Yes	Yes	Yes	No	No	No
California	No	No	No	Yes	Yes	Yes	Yes	Yes	No	No
Arizona	No	No	No	No	Yes	Yes	Yes	Yes	Yes	No
Mississippi	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes

*Source: Hayward Alker, Jr., "Statistics and Politics," in Seymour Martin Lipset, ed., *Politics and the Social Sciences* (New York: Oxford University Press, 1969), Table 2, p. 276.

1964 presidential contenders. Each voter (here residents of particular states) is asked to indicate the five candidates he or she likes most, and the perfect proximity scale means that all voters choose only adjacent candidates. If the left-right candidate order was not known, one would permute the candidates until each person chooses only adjacent candidates. The candidates are thereby ordered from left to right. The restriction to naming five candidates is not essential to the model; if the candidate space is unidimensional then the voters should like only adjacent candidates regardless of the number each names. The important point is that this is not a cumulative process: to be a liberal does not mean you must like all the candidates more conservative than you. A person can accept one candidate without accepting all more conservative (or liberal) candidates.

Multiparty coalitions can also be expected to follow proximity notions. Weisberg has shown that the cabinets of the French Fourth Republic form a proximity scale in that 17 of the 19 cabinets include only parties which are adjacent on a left-right dimension.[18] One of the two exceptions involves the absence of the MRP from the Mendes-France government, an unusual case in which Mendes took the exceptional step of negotiating with individuals rather than with the parties in forming his cabinet but the MRP party blocked his naming two MRP members to his cabinet.

When would proximity scales be found? The argument is that proximity scales will occur when two conditions are met simultaneously.[19] There must be some variance in individual preferences, so that not everyone most prefers a maximum (or a minimum) of the dimension. Additionally, the individuals must only indicate which alternatives they consider acceptable, rather than indicating which element of a pair comparison they prefer; for example the radical may prefer a moderate change to the status quo but still not consider the moderate change sufficiently useful to support it. Thus proximity scales may be expected where the *direction* of social change is at issue and where radicals oppose liberal reforms which ameliorate the situation without restructuring society. If either of these conditions is not met, Guttman scales (or their generalizations for nondichotomous and/or multidimensional data) will be more likely.

[18] Herbert Weisberg, "L'etude comparative des scrutins legislatifs," *Revue Francaise de Sociologie,* 12 (April–June 1971), 151–176. See also Robert Axelrod, *Conflict of Interest* (Chicago: Markham, 1970) and William Riker and Peter Ordeshook, *An Introduction to Positive Political Theory* (Englewood Cliffs, N.J.: Prentice-Hall, 1973), chapter 7.

[19] For a proof of these results see Weisberg, "L'etude comparative des scrutins legislatifs."

The considerable success of Guttman scaling cannot be lightly dismissed. The suggestion here is only that there exists a class of situations which can be treated as unidimensional even if Guttman scaling finds no cumulation. Yet those models are related. For a proximity scale, each person accepts adjacent alternatives, but that is also true of the circular and Guttman scales. The circular scale is the most general, the proximity next, and the Guttman scale is the most restrictive. For example, with dichotomous data there are 16 possible response patterns with four variables, of which 14 fit the circular model, 11 the proximity scale, and only 5 satisfy a Guttman scale. Thus we should always expect the circular and proximity scales to fit data better (or at least no worse) than the Guttman model. This means that the restrictive Guttman model would be employed unless the other models do a significantly superior job in fitting the data. Of course, the choice of model can be motivated by the nature of the underlying substantive process regardless of empirical fit, as in using a circular representation whenever the extremes unite in favor of a proposal that the center opposes.

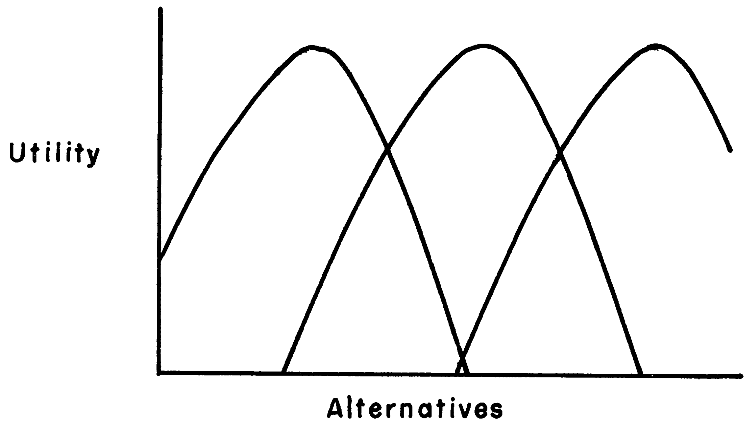
Much depends on the purpose of the scaling endeavor. If one simply wishes to ascertain the nature of the substantive process, it is important to employ the widest possible definition of unidimensionality. There is no gain in describing a process as two-dimensional if it is unidimensional under a broader conception. Often, however, one scales to construct analytic measures, as when scaling legislative votes to obtain behavioral indices which can be correlated with constituency attitudes or characteristics. One then might want to obtain several measures of behavior even under a unidimensional process. For example, one might seek to measure extremeness as well as ideological position. Thus the choice of what is a unidimensional representation may depend on the purpose of the analysis.

Antiland: An Extremist World with Negative Peaks

Scaling models can have opposites, models which are their duals but with directions reversed. New scaling models arise when we consider these mirror image duals of the conventional models. This provides a series of negative models with distinctive substantive implications.

Scaling models for preference data assume that everyone has a point of *maximum* preference, known as an "ideal point." The person would rank order alternatives in terms of preferences, which means the person would like alternatives more the closer they are (or the closer the person thinks they are) to his or her ideal. The preference function is then single-peaked, as are the curves in Figure 5. For each person the greatest utility is being given by one

FIGURE 5
Single-Peaked Preference Functions



alternative and less utility is obtained as alternatives are further from the most preferred alternative. Single-peaked preferences constitute a common definition of unidimensionality.[20]

The mirror image concept is the anti-ideal. Each person has a point of minimum preference, an anti-ideal or a negative ideal point. The person intensely dislikes that alternative and dislikes other alternatives more the closer they are to that alternative. Unidimensional preference functions in this model are single-caved rather than single-peaked.[21]

This negative ideal model would apply to a situation where people are dissatisfied with moderate solutions and believe that sharp change is vital regardless of its direction. People dissatisfied with the conduct of a limited war may have a negative ideal at the status quo and have as their first choices either immediate withdrawal from the war or military victory (Figure 6).[22]

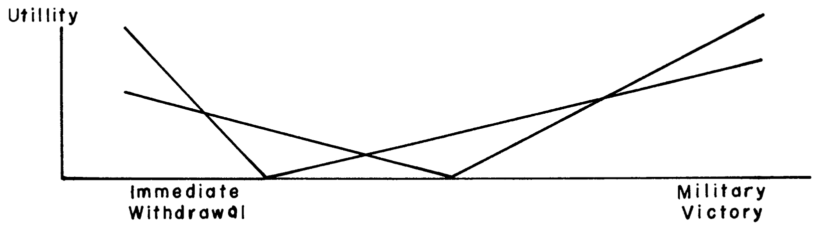
[20] Clyde Coombs, *A Theory of Data* (New York: John Wiley and Sons, 1964), chapters 5 and 9.

[21] The anti-ideal has been operationalized in Carroll and Chang's PREF-MAP computer program. See J. Douglas Carroll and Jah-Hie Chang, "Relating Preference Data to Multidimensional Scaling Solutions via a Generalization of Coombs' Unfolding Model," (Bell Telephone Laboratories, Murray Hill, N.J., 1967).

[22] A more realistic model for explaining preferences with regard to the Viet Nam war would be two-dimensional, with one direction dimension (ranging from withdrawal to victory) and one speed dimension (ranging from seeking an immediate solution to approval of gradual solutions). Many of the public favored a fast solution regardless of its direction, weighting the speed dimension much more heavily than the direction dimen-

FIGURE 6

Hypothetical Single-Caved Preference Functions on a Viet Nam War Involvement Scale



When the populace so despairs of moderate solutions that it demands change regardless of its direction, the negative ideal model is appropriate. The preference functions of Figure 6 would require multiple dimensions for scaling if normal scaling models were employed, but a single dimension would suffice for the negative ideal model. The fallacy would be in not using a unidimensional negative model when it is appropriate.

Coombs's unfolding analysis is the technique used for scaling preference orders under single-peakedness. If preferences are single-peaked, then no one would prefer both immediate withdrawal and military victory over intermediate solutions to a foreign war. To generalize, with unidimensional preferences the middle points on the dimension would never be selected as a person's last-place choice. Only the two end items of the dimension would be picked as last-place choices, as should be apparent from Figure 5. Inspection of Figure 6 suggests the corresponding conditions for the negative ideal model. If preferences are single-caved, no one would prefer both immediate withdrawal and military victory *less* than intermediate solutions. That is, with unidimensional preferences the middle points would never be selected as a person's first-place choice. Only the two end items of the dimension would be picked as first-place choices.[23] This is the reverse of the statement for

sion. Yet the negative ideal model does capture part of reality, since some citizens probably did have preference functions like those of Figure 6. To test the models one would have to ask people's preferences over a set of alternatives with time held constant, such as asking people if they prefer noninvolvement, limited involvement, or full involvement.

Figure 6 shows preferences over a limited set of alternatives in the sense that people's preferences might not continue to be more favorable as yet more extreme alternatives are raised. Even the hawks might give lower utility to the more extreme possibility of military victory through use of nuclear weapons. This suggests that the preference functions can only be considered over the range of realistic alternatives. The dimension may have to be bounded to apply the negative model.

[23] Consequently the negative model should be investigated whenever only two alternatives receive the bulk of first-place choices.

single-peaked preferences, and similar reversals occur throughout the conditions for unidimensionality under single-cavedness. As a result, to scale under the negative ideal model one need only apply conventional unfolding analysis to the reverse of each person's preference order. Negative models require no new scaling techniques; they require only stepping away from one's data to realize the data are the mirror image of conventional data.

We use scaling to describe and understand a complex political reality. Geometric models may be useful in this endeavor, but no single model should be expected to capture all of that reality. Multiple models may be appropriate, each describing part of reality. Each model draws attention to some aspect of the data: a circular model where extremists of the left and right behave similarly, a proximity model where extremists oppose desirable solutions that are too moderate, a negative model where people are dissatisfied with centrist solutions. Each model goes beyond conventional left-right notions without requiring us to posit the existence of multiple dimensions. Fairly complex attitudinal data can be ideologically unidimensional if our conception of unidimensionality is sufficiently general. Whether any particular example really fits these models is not important here, so long as these examples begin to suggest why such unconventional models require consideration.

Timeland: An Unseen World of Traces

The previous sections emphasized that sets of data which appear to be multidimensional can be unidimensional given the proper scaling model. The remaining sections switch to the opposite point: what seems unidimensional may sometimes be better understood as multidimensional.

One of the simplest interpretations of unidimensionality is a natural order. Objects can often be readily ordered but whether attitudes towards those objects are based on that ordering is an empirical question. Stimuli may be unidimensional with respect to some property, but preferences toward them need not be based on that dimension. For example, chocolates can be arrayed in terms of their sweetness, but preferences for chocolates may not be based on that natural order since many people prefer both sweet chocolate and bitter-sweet to the melange created by blending the two together. Natural orders may be irrelevant to preference behavior.

Time is one of the most important natural orders. Time is usually ignored in scaling, and we act as if all the observations were collected simultaneously even when that is untrue. But instead of viewing time as an unimportant complication, we can view time itself as the dimension. Time provides the backdrop against which objects develop. If time is viewed as a dimension, development over time can be scaled. Yet the argument above is that development need not be unidimensional, even if time is the relevant natural order.

Developmental processes can be described by a variety of models at different levels of complexity. The simplest notion is that development is a series of stages which follow in a predictable manner. Countries may pass through certain stages of economic development in an ever upward direction. The stage model breaks down for developmental processes in which acquisition of a new trait does not require deletion of all previous traits. Children may acquire new abilities in a specific order with complex skills not being acquired until prerequisite ones are in place, but acquisition of a new ability does not entail deletion of earlier traits.[24] A more complex model would permit ordered acquisition with deletion. A country moving up to a certain cultural level may retain most of its preceding cultural forms while deleting its most primitive cultural attributes. The deletion order may differ from the acquisition order, with an early trait lasting longer than some later acquired trait.[25]

Leik and Matthews have termed ordered acquisition with deletion a "developmental scale."[26] They suggest an example in terms of leisure time

[24] Snow has applied such a cumulative development model to political development in Latin America, with freedom of political organization for autonomous groups being the easiest trait to acquire and a modern bureaucracy the most difficult trait. See Peter Snow, "A Scalogram Analysis of Political Development," *American Behavioral Scientist,* 9 (March 1966), 33–36.

[25] A time model could actually be circular in the case of cyclical phenomena. For example, if one were to scale over time several indicators of interest in presidential politics (public interest as measured in surveys, newspaper column inches devoted to the campaign, and so on), interest would likely rise as the election approaches, peak at election day, fall sharply thereafter, and gradually increase as the next election approaches. If time were measured since the last presidential election, interest would fit a circular dimension. Such a cyclical development pattern would be violated if interest in presidential politics had a short-term surge around the congressional off-year elections.

Coombs and Smith present an even more general developmental model in which acquisition and deletion processes are independent. The traits are acquired in a fixed order and they are deleted in a fixed order, but the two processes are separate. Thus after acquiring traits A and B, some individuals may delete A to be left with trait B while others would acquire C to have traits A, B, and C. See Clyde Coombs and J. Keith Smith, "On the Detection of Structure in Attitudes and Developmental Processes," (Michigan Mathematical Psychology Program, 1972). The two-dimensional conjunctive scaling technique they suggest for this model is described in Clyde Coombs, *A Theory of Data,* chapter 12.

[26] Robert Leik and Merlyn Matthews, "A Scale for Developmental Processes," *American Sociological Review,* 33 (February 1968), 62–75.

TABLE 2

Hypothetical Scale of a Person's Leisure Time Activities

	Age											
Activity	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50	51–65	66–80
Tricycle	Yes	No	No	No	No	No	No	No	No	No	No	No
Marbles	Yes	Yes	No	No	No	No	No	No	No	No	No	No
Bicycle	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No
Bridge	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Bowling	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No
Squash	No	No	No	No	No	No	Yes	Yes	Yes	Yes	No	No
Knitting	No	No	No	No	No	No	No	Yes	No	No	No	No
Cribbage	No	No	No	No	No	No	No	No	No	No	No	Yes

TABLE 3

Developmental Scale of Votes at 1852 Democratic National Convention*

	Ballot											
Candidate	1	5	10	15	20	25	30	35	40	45	48	49
Cass	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Douglas	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Buchanan	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No
Marcy	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes	No
Pierce	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes

*Yes indicates at least 29 of the 288 votes; No indicates 28 or less. Based on Richard Bain and Judith Parris, *Convention Decisions and Voting Records*, 2nd ed., (Washington, D.C.: The Brookings Institution, 1973), Appendix C.

activities. People adopt given leisure activities at certain times of their lives, continue them for a period, and eventually may drop them. Some activities may be continued throughout one's life, while others tend to be confined to a particular age period (youth, middle age, or old age). Table 2 shows a hypothetical person's developmental scale of leisure activities. The developmental scale algorithm begins with the order of time points known. Each trait is then checked to determine if it is possessed only during adjacent time points. The developmental pattern is violated if the trait is acquired, deleted, and then reacquired.[27]

Table 3 applies this logic to presidential nomination ballots at the 1852 Democratic national convention.[28] The candidates gained and lost strength along a time dimension. Cass and Douglas were strong throughout the balloting, Buchanan faded early, Marcy started late but lost, and another late starter won—Franklin Pierce won the nomination virtually unanimously on the 49th ballot even though he was still under 10% of the votes on the 35th ballot. Never did a candidate fall out of competition and then return, the pattern which would violate a developmental model.[29]

[27] The developmental scale is a proximity scale (or a Guttman scale if deletion does not occur) in which the order of alternatives is known in advance to be the time order. Thus the developmental scale is very restrictive since the order of alternatives is fixed rather than being chosen to maximize fit with the data.

[28] Data are from Richard Bain and Judith Parris, *Convention Decisions and Voting Records,* 2nd ed., (Washington, D.C.: The Brookings Institution, 1973), Appendix C.

[29] The use of a 10% cutoff is an arbitrary device to permit the developmental scale to be displayed in the proximity scale mode. An alternative procedure would be to plot for each candidate the number of votes obtained (on the ordinate) against the ballot number

Yet this unidimensional result is not a necessary outcome. A vivid contrast is presented by the 1924 Democratic nomination. John W. Davis began with only 31 of the 1098 votes, peaked at 129.5 on the 23rd and 24th ballots, fell to a low of 40.5 votes on the 58th ballot, and then went back over the 10% mark on the 95th ballot and won the nomination on the 103rd. Development need not follow a linear pattern. Indeed the most interesting cases to study might well be those like the 1924 nomination where development was not unidimensional.

In scaling developmental processes, our interest is in whether there is an orderliness to the development. We may not find regularized stages of development; indeed we may find that development goes back and forth without advancement. We assume that time is the dimension against which development occurs, but the developmental scaling fails if that assumption is in error.[30]

Natural orders, such as time, can sometimes form the basis for dimensions. Temporal evolution of attitudes can occur, such as in the argument that people become more conservative as they age.[31] Yet preferences and behavior may not always be based on a preconceived natural order, and even development need not be monotone with time. The existence of a natural order does not suffice to guarantee that a unidimensional perspective is appropriate.

Cumulativeland: A Unidimensional World without Single-peakedness

To what extent do different scaling models employ compatible notions of unidimensionality? Niemi and Weisberg have shown that two of the most

(on the abscissa). A developmental model would suggest single-peaked shapes for these plots, as in Figure 5, whereas deviation from developmental patterns would yield multipeaked curves. The actual vote totals for the 1852 convention data are much more bumpy than suggested by Table 3. For example, Cass started with 116 votes, slipped to 33 by the 30th ballot, but then went all the way up to 131 votes on the 35th ballot.

[30] Time processes can also be generalized by defining the abstract characteristics of time sets as in Thomas Windeknecht, *General Dynamic Processes* (New York: Academic Press, 1971), pp. 13–16. Monetary values would qualify as a time set in that they are nonnegative real numbers with addition of zero not changing their values, so similar procedures could be used to test whether preferences toward spending various amounts of money on government programs are consistent with the money dimension.

[31] See the discussion of time-related attitude change in M. Kent Jennings and Richard Niemi, "Continuity and Change in Political Orientations," paper delivered at 1973 American Political Science Association meetings, New Orleans.

common meanings are in conflict—and by implication that different techniques may yield very different types of dimensions.[32]

The first meaning they consider is single-peakedness. In attitude theory a prime interpretation of unidimensionality involves everyone having single-peaked preferences over a common dimension as in Figure 5. Single-peakedness plays an important role in formal theory. Black, for example, shows that the paradox of voting cannot occur if all individuals have single-peaked preferences.[33] Thus social choice is rational in a unidimensional culture. Clyde Coombs developed "unfolding analysis" to test preference order data for single-peakedness.[34] However preference orders are often not available, so we would like to be able to test other types of data for single-peakedness.

A second meaning of unidimensionality is that of cumulation. Cumulation occurs if people accept a proposal only if they accept all weaker proposals. Guttman scaling ascertains the fit with unidimensional cumulation. Guttman scales are spoken of as unidimensional, and Guttman scaling is thus a procedure for testing unidimensionality of dichotomous data. Indeed it may be the most common test for unidimensionality in attitude research.

The question is whether these two meanings of unidimensionality are equivalent. If single-peakedness connotes unidimensionality and if a Guttman scale is unidimensional, can we use Guttman scaling to test for single-peakedness? Niemi and Weisberg provide a counter-example to demonstrate that such an identity is fallacious. Figure 7 shows their example. The acceptances fit a cumulative Guttman scale perfectly, assuming that each person will accept items only if they provide the indicated minimum of utility. However not all preferences are single-peaked. The three preference functions have been devised so that single-peakedness will not hold for all individuals regardless of any reordering of the alternatives along the horizontal axis. Thus Guttman scales do not guarantee single-peakedness.[35]

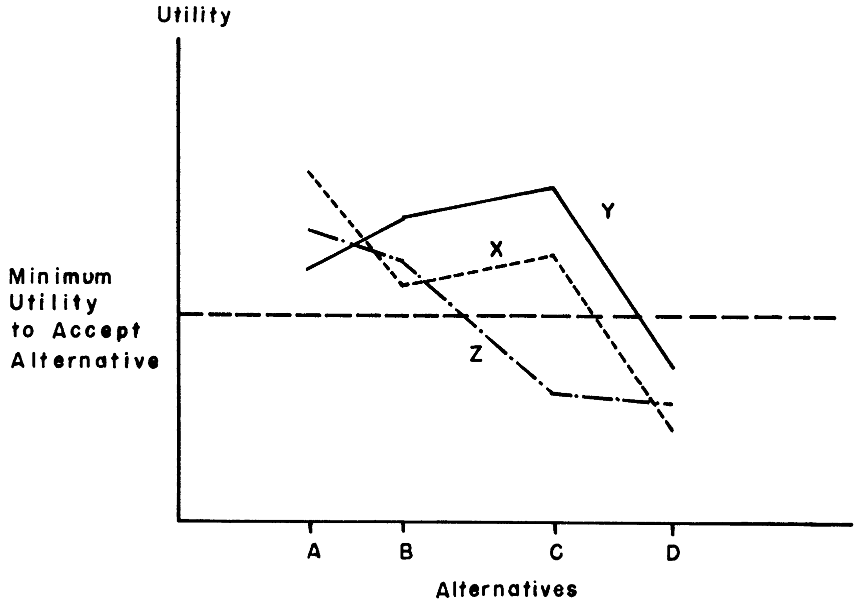
[32] Richard Niemi and Herbert Weisberg, "Single-Peakedness and Guttman Scales: Concept and Measurement," *Public Choice,* forthcoming.

[33] Duncan Black, *The Theory of Committees and Elections* (Cambridge: Cambridge University Press, 1958).

[34] Clyde Coombs, *A Theory of Data,* chapter 5.

[35] A complete proof also requires demonstration that Guttman scales do not imply single-peakedness when individual responses are based on choosing which of a pair of alternatives the person prefers more. The proof is as simple as that given here, but the reader is referred to Niemi and Weisberg, "Single-Peakedness and Guttman Scales" for the counter-example. They also push the argument further to demonstrate that a person's Guttman scale score is not indicative of the person's ideal preference point on the dimension. Note that it has only been shown that Guttman scalability does not imply single-peakedness; under appropriate conditions, single-peakedness may still imply Guttman scalability.

FIGURE 7
Preference Functions over a Single Dimension*



X:	Yes	Yes	Yes	No
Y:	Yes	Yes	Yes	No
Z:	Yes	Yes	No	No

Response Patterns

*Taken from Richard Niemi and Herbert Weisberg, "Single-Peakedness and Guttman Scales: Concept and Measurement," *Public Choice,* forthcoming.

It would be folly to interpret this case as showing that some other scaling technique (such as factor analysis or multidimensional scaling) should be used in preference to Guttman scaling. The problem is in the type of data available: single-peakedness can be tested only with preference order data, however difficult they are to obtain.[36] Only unfolding analysis treats "single-peakedness"; other scaling techniques just test whether the data conform to a single-parameter ("unidimensional") model.

[36] The difficulty found in Figure 7 might not occur if the data were multicategory rather than dichotomous, but only complete preference order data suffices to guarantee the detection of multipeaked preferences.

Factorland: A Mechanized World of Extra Dimensions

Some scaling techniques are usually considered in a complex multidimensional context. To understand them it is necessary to determine what they treat as unidimensional, so that the choice between alternative procedures can be based on which notion of a dimension is most appropriate. Nonmetric multidimensional scaling and factor analysis are two of the most important multidimensional techniques, but their understandings of unidimensionality are quite disparate.

Nonmetric multidimensional scaling, as developed by Shepard and Kruskal, seeks to obtain a spatial representation of data such that the more similar are a pair of stimuli (such as, the higher is their correlation), the closer together are their points in the space.[37] That is, the interpoint distances are monotone with the corresponding data values. A single dimension is found if objects can be uniquely ordered with the most different at opposite ends and with smaller distances representing smaller differences. The dimension orders the objects along a continuum according to how different they are from one another. More than one dimension is obtained if the distance relations between points cannot be satisfactorily accommodated with a single dimension.

The principal component factor analysis model instead represents correlations by the cosines of the angles between vectors.[38] For example, a correlation of zero would be displayed by 90° separation between the lines connecting the variables' points with the origin. A single dimension is found if variables covary perfectly, which is to say if the variables are identical to one another except for linear transformations. The dimension shows the extent to which the variables are saturated with the same common element. Variables at opposite ends of a dimension are opposites of one another.[39] Table 4 summarizes some of the differences in the two scaling models.

[37] See J. Kruskal, "Multidimensional Scaling by Optimizing Goodness of Fit to a Nonmetric Hypothesis," *Psychometrika,* 29 (March 1964), 1–27. The essential concepts are similar to those of smallest space analysis as developed by Guttman and Lingoes; see Louis Guttman, "A General Nonmetric Technique for Finding the Smallest Coordinate Space for a Configuration of Points," *Psychometrika,* 33 (December 1968), 173–192.

[38] The component model of factor analysis is considered throughout this section rather than the common factor model which represents correlations by scalar products (cosines multiplied by the product of the lengths of the two vectors). For a more general discussion of the differences between multidimensional scaling and factor analysis, see George B. Rabinowitz, "An Introduction to Nonmetric Multidimensional Scaling," *American Journal of Political Science,* forthcoming.

[39] Multiple dimensions would be obtained only if the variables can be partitioned into fairly separate clusters. Trivial factors based on error in the data would be excluded by Kaiser's criterion of using only factors which explain at least one unit of variance.

TABLE 4

Differences between Multidimensional Scaling and Factor Analysis

	Multidimensional Scaling	Factor Analysis
Spatial representation:	distances	cosines of angles
Unidimensionality:	unique ordering with interpoint distances monotone with data values	perfect covariation
Opposite ends of dimension:	most different stimuli	variables that are opposites
Located at same point:	no differences between the stimuli	perfect covariation of the variables
Type of dimension:	difference continuum	saturation factor

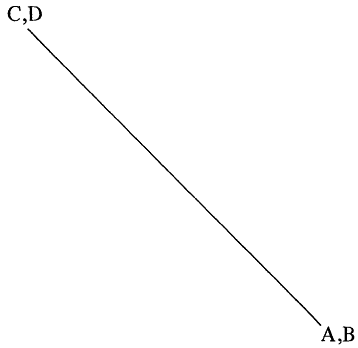
These are two different spatial representations which are appropriate in distinct situations. If the data consist of distance measures, then multidimensional scaling is clearly required. The cosine formulation of factor analysis might be essential in other instances, as in dealing with numeric attribute data. However there are some cases, such as the analysis of correlation coefficients, in which either display is possible. In some situations the two techniques will yield different results, so there must be a choice made between the two representations. Specifically the question to be addressed is how the techniques differ in the analysis of correlation statistics. There has been little systematic study of this topic, though a preliminary set of conjectures can be advanced.

A preliminary difference is that factor analysis can yield more dimensions than multidimensional scaling.[40] This is partly due to the fact that multi-

[40] Factor analysis also can yield more dimensions than Guttman scaling; see Aage Clausen, *Policy Dimensions in Congressional Roll Calls* (Unpublished Ph.D. dissertation, The University of Michigan, 1964). This is due to the choice of correlation coefficient for the factor analysis; see Herbert Weisberg, *Dimensional Analysis of Legislative Roll*

FIGURE 8

Multidimensional Scaling of Independent Sets of Variables



Correlations

	A	B	C	D
A	–	1.0	0.0	0.0
B		–	0.0	0.0
C			–	1.0
D				–

dimensional scaling only takes account of the ordinal features of the data while factor analysis considers their interval values. The simplest example is that distances between three objects can always be represented in one dimension using a nonmetric solution,[41] but two dimensions may be required for a metric fit.[42] Also, factor analysis can involve an extra dimension because it employs an origin to permit angle calculations, while the interpoint distances considered in multidimensional scaling do not depend on the placement of an origin. Thus Figure 8 shows that multidimensional scaling requires only a single dimension to portray the distinction between two sets of points without an origin, while factor analysis of the same data (Figure 9) requires two dimensions since the independence of the two sets of points can only be shown with respect to an origin.[43] One more dimension may be

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Calls, chapter 5. The differences described in the text may similarly be due to analysis of the wrong type of correlation coefficient with one of the techniques, but the conclusions still show some important relationships between the techniques.

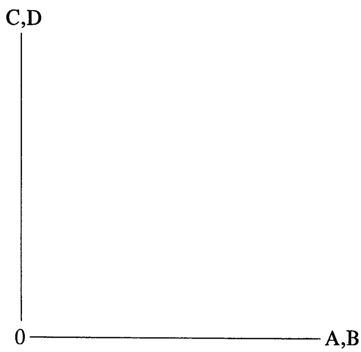
[41] Let the points with the greatest distance be at opposite ends of the dimension. Place the third point closer to the point with which it has the least distance. Necessarily then the middle distance is between the third point and the other point.

[42] For example, if A and B are two units apart, A and C are three units apart, and B and C are four units apart, then two dimensions are required for a metric fit. A nonmetric solution would simply place B and C at opposite ends of a single dimension with A positioned on that dimension closer to B than to C.

[43] In these and later figures, at least two variables are always located at the same point with perfect correlations among them. Multidimensional scaling can obtain trivial solutions when there are small numbers of variables with many equal correlations, so the perfect correlations are used here to force meaningful multidimensional scaling solutions.

FIGURE 9

Factor Analysis of Independent Sets of Variables



Correlations

	A	B	C	D
A	–	1.0	0.0	0.0
B		–	0.0	0.0
C			–	1.0
D				–

necessary for factor analysis since one more point—the origin—is to be scaled.

The extra dimension produced by factor analysis is essential when locating individuals in the space. If individuals were assigned factor scores on the basis of Figure 9, each individual would obtain two scores which would be independent of one another. Figure 8 is based on the same data, but there is no way of scoring individuals in multidimensional scaling to yield the same information. When individuals are to be located in the space, factor scores are more appropriate than multidimensional scaling of correlations.

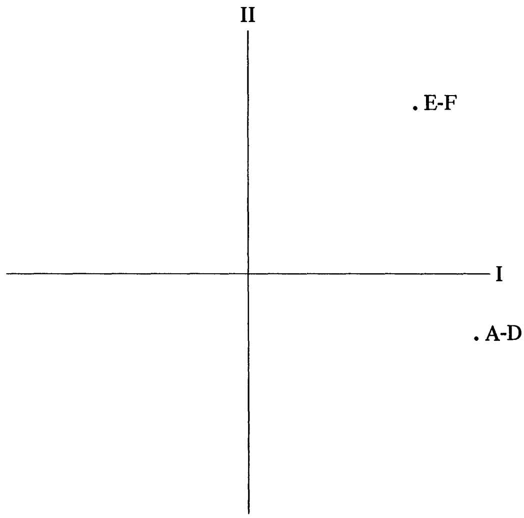
But when does factor analysis yield one more dimension than multidimensional scaling, and what is the meaning of that dimension? Three cases must be distinguished. The conditions defining the cases will not be specified in precise terms here, but the differences in the relationship between factor analysis and multidimensional scaling will be emphasized.

The first case is when all variables have the same direction so that virtually all correlations are positive. Say there are perfect correlations among variables A–D and among variables E–F but only .50 correlations between the two sets. The multidimensional scaling solution for these correlations is unidimensional with the two clusters of points at opposite ends of the dimension. The factor solution of Figure 10 instead employs two dimensions.[44] The clusters are separated most on the second axis, and that axis best captures the multidimensional scaling solution. Since the variables have the same direction, they all have positive loadings on the first factor component.

[44] Figures 10 and 11 give unrotated principal axes. Oblique rotated solutions would place one axis through the A–D cluster and another through the E–F cluster, with the angle between the axes being the angle whose cosine is the between-set correlation.

FIGURE 10

Unrotated Factor Analysis of Positively Correlated Sets of Variables



Correlations

	A	B	C	D	E	F
A	–	1.0	1.0	1.0	0.5	0.5
B		–	1.0	1.0	0.5	0.5
C			–	1.0	0.5	0.5
D				–	0.5	0.5
E					–	1.0
F						–

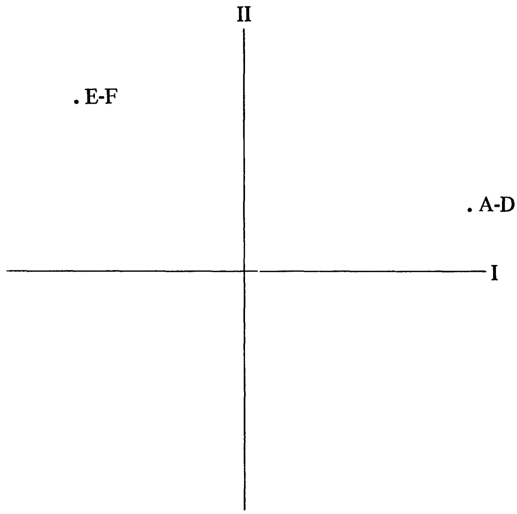
All variables have high loadings on the most important principal axis, which is a general component that emphasizes what the variables share in common. Rather than order the variables in terms of how different they are from one another, the first component shows the extent to which each item is saturated with the common factor.

The second case is when one set of variables has the opposite direction of another set of variables, so that the between-set correlations are essentially negative. Again let there be perfect correlations among variables A–D and among variables E–F, but let the between-set correlations be –.50. The multidimensional scaling solution remains as above, but the factor analysis principal components are now those of Figure 11. The clusters are separated most on the first principal component. The opposite directions of items is captured by the first principal axis, and that is the main difference between items which would dominante a multidimensional scaling space. The second factor component is of substantially less importance.

The final case is when there are more than two sets of variables with zero or negative correlations between the various sets. Figure 12 gives an example in which there are sets of negatively correlated variables which are independent of other sets of negatively correlated variables. The two techniques yield identical solutions for this case. As many dimensions are required to capture

FIGURE 11

Unrotated Factor Analysis of Negatively Correlated Sets of Variables



Correlations

	A	B	C	D	E	F
A	–	1.0	1.0	1.0	–0.5	–0.5
B		–	1.0	1.0	–0.5	–0.5
C			–	1.0	–0.5	–0.5
D				–	–0.5	–0.5
E					–	1.0
F						–

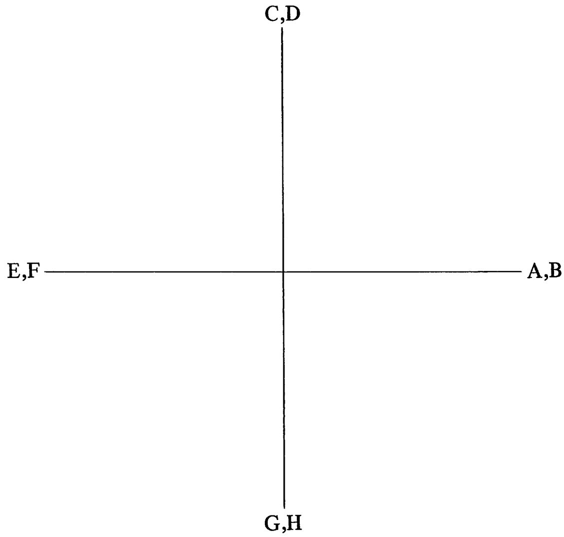
the differences between the variables in multidimensional scaling as are necessary to give angular representations to the correlations in factor analysis. This is the case where the two techniques report the same dimensionality.

Altogether, the first factor analysis principal axis gives two types of information. It shows the item directions and, ignoring signs, it shows how saturated the variables are with a common factor. The items are not ordered in terms of their differences from one another but in terms of how much they partake of the central core. This latter piece of information is actually a part of classical statistics. When we distinguish between independent and dependent variables, we use regression to determine how much of the variance of the dependent variable can be accounted for by the independent variables. When the distinction between independent and dependent variables is inappropriate (as in analyzing the structure of a party system), factor analysis is used to determine how much of the variance of the total set of items can be accounted for by a single hypothetical construct–the first principal component–and to what extent each variable measures that common element. Thus the component model of factor analysis provides a classical solution to a statistical question, while multidimensional scaling has no analogue to this information.

How will multidimensional scaling results differ from factor analysis?

FIGURE 12

Multidimensional Scaling and Factor Analysis of Independent Bipolar Structure



Correlations

	A	B	C	D	E	F	G	H
A	–	1.0	0.0	0.0	–1.0	–1.0	0.0	0.0
B		–	0.0	0.0	–1.0	–1.0	0.0	0.0
C			–	1.0	0.0	0.0	–1.0	–1.0
D				–	0.0	0.0	–1.0	–1.0
E					–	1.0	0.0	0.0
F						–	0.0	0.0
G							–	1.0
H								–

Comparison of Figures 10–12 suggests a complex answer. When the items have the same direction, the multidimensional scaling solution will correspond to the factor space deleting the first general component. When the items have different directions, the first principal components of the two techniques will be similar. Indeed when there are sets of items with opposite directions which are independent of other such sets, the two solutions will be similar for all dimensions. Other differences between the two techniques seem to have little impact on the solutions.[45] This statement should be regarded as a conjecture based on a set of simple data structures. These results would not hold exactly in the analysis of real data since the two techniques handle error differently, but tests of the conjecture on real data are very encouraging.

This conjecture has been tested on correlations between attitudes toward presidential contenders in the 1972 American national election survey of the University of Michigan's Center for Political Studies.[46] Feelings toward Republican and Democratic leaders were negatively correlated, and both techniques focused most attention on that distinction. Consequently there was a .998 correlation between the first principal axes obtained by multidimensional scaling and factor analysis.[47] To test the conjecture further, scores given to some candidates were reversed so that a 100° thermometer score would always mean a liberal response.[48] That is, variables with negative loadings on the first factor of the original analysis were reversed so all items would have the same direction. The analyses were performed a second time on the revised data. The correlation between the first principal axes fell to −.004; only half of the variation on the first factor could be accounted for by linear prediction from the three multidimensional scaling dimensions. Yet the second factor had a .996 correlation with the first multidimensional scaling

[45] Factor analysis represents data by scalar products between vectors while multidimensional scaling employs interpoint distances. Yet even this difference will not cause sharp discrepancies in solutions, except that the scalar product representation necessitates an origin. For example, the scalar product between points *j* and *k* can be written in terms of the distance d_{jk} between the points and the distances, d_j and d_k, of each point from the origin: $-.5\ (d_{jk}^2 - d_j^2 - d_k^2)$ which equals $c - .5d_{jk}^2$ if all variables are a constant squared distance *c* from the origin. According to that result, the scalar products would be monotone with the squared interpoint distances which are obviously monotone with the unsquared interpoint distances. Consequently nonmetric analysis of scalar products and interpoint distances will differ only as items are unequal distances from the origin.

[46] The data are the candidate feeling thermometers from the preelection survey. They were made available by the Inter-University Consortium for Political Research. Neither the original investigators nor the Consortium bear any responsibility for the analysis or interpretations presented here.

[47] The correlation reported here is the correlation between each variable's loading (projection) on the first factor component with its loading on the first multidimensional scaling dimension. Note also that the multidimensional scaling solution employed here is a principal axis rotation around the centroid of the points. That rotation possesses statistical meaning since it can be shown to account for a maximum proportion of the variance of the points with each successive orthogonal dimension.

[48] The reversing of candidate thermometers is intended only to illustrate the difference between the two analysis techniques, and not as a substantively useful procedure.

dimension, and the third factor had a .870 correlation with the second multidimensional scaling dimension. When all items have the same directions, the first factor component is unrelated to the multidimensional scaling space while the later factors correspond closely to the multidimensional scaling solution. Thus both parts of this test confirm the conjecture developed above.

Factor analysis can produce extra dimensions which serve particular purposes: indicating the extent to which variables share a common core, providing the item directions, and giving a space for scoring individuals. But what is more fundamental is that factor analysis and multidimensional scaling involve different types of dimensions. In some instances they provide identical information, but when they do not the factor analysis of correlation matrices provides more information and can be used for multidimensional scaling purposes. Multidimensional scaling seems appropriate only when it yields the same dimensionality as factor analysis or when a distance model is required.[49] Overall, these two multidimensional procedures differ in their interpretations of unidimensionality, so considerable care is essential in the choice between them.

Dimensionland: An Analogue World of Games

The existence of different forms of unidimensionality and nonequivalent types of dimensions permits no simple conclusions as to the true nature of unidimensionality. This should not be interpreted as questioning the scientific utility of scaling, but rather should emphasize the importance of considering a wide variety of alternative scaling models. Geometric models are employed to facilitate understanding the nature of political reality, so limiting the breadth of geometric models is unnecessarily restrictive. This perspective implies avoiding resort to multidimensional explanations when the data are unidimensional according to some other applicable meaning of unidimensionality, while still adopting multidimensional measurements when models permitting unidimensional fit are inappropriate. This is basically a plea for

[49] If the items have meaningfully different directions which should not be corrected (as when scaling attitudes toward presidential contenders), the multidimensional scaling solution is quite proper. When the items have artificially opposite directions which are not intended to be the focus of the analysis (as when scaling legislative roll call votes—which occasionally have yes as a liberal response and other times no as a liberal response—with an interest in finding the underlying dimensional structure without regard to item direction), the multidimensional scaling solution is useless unless the items are properly directed in advance and even then the analysis in the text suggests that factor analysis would be more useful.

flexible use of scaling models, but it has some intriguing philosophical implications that merit explicit attention.

First, different scaling models permit representation of different aspects of reality, so a complete portrayal of political reality may require the use of several scaling models and multiple images. This contradicts the simplistic notion that objective scientific methods can yield unique solutions. As Abraham Kaplan argues, "Truth may be one, but if so, this proposition holds at best only for literal statements; there is no limit to the metaphors by which we can effectively convey what we know. . . . If the model is not conceived as picturing reality, we can make good use of several models, even if they are not compatible with one another."[50] Science thus becomes the search for a multiplicity of partial images. This does not signify a shortcoming of scaling methodology, but just illustrates the role of models in science. Metaphors—geometric as well as other types—are used in scientific inquiry, but all of the real world behavior can never be captured in a single metaphor. Multiple models are useful, and it is worth being reminded that no single representation is ever identical to the world one seeks to describe.

Second, our limited set of familiar and usable scaling models must inevitably restrict our ability to describe the political world. Our store of models determines our potential for understanding political reality. This underlines the dependence of scientific conclusions on proper concept development. Science describes the world, but only within the context that science prescribes. Reality cannot be distinguished from the manner in which we study it. This view is similar to the Sapir-Whorf hypothesis of linguistic determinism operating on culture. Linguistic relativity states that "all observers are not led by the same physical evidence to the same picture of the universe, unless their linguistic backgrounds are similar."[51] The consequence is that "human beings do not live in the objective world alone, . . . but are very much at the mercy of the particular language which has become the medium of expression for their society."[52] Substitute the term "scientific concepts" for "linguistic backgrounds" and "language" in these statements and one has the result that the set of concepts available to science at any point of time must inevitably limit the conclusions that science can make. Furthermore, differences in scientific concepts yield different views of the real world.

[50] Abraham Kaplan, *The Conduct of Inquiry,* p. 287.

[51] Benjamin Whorf, as quoted in John B. Carroll, ed., *Language, Thought, and Reality: Selected Writings of Benjamin L. Whorf* (Cambridge, Mass.: MIT Press, 1956), p. v.

[52] Edward Sapir, *Selected Writings in Language, Culture, and Personality,* edited by David Mandelbaum (Berkeley: University of California Press, 1964), p. 162.

Finally, the very existence of a knowable reality may well be questioned. The inhabitants of Abbott's Flatland live in a two-dimensional world, and the discovery of more dimensions has minimal effect on their existence. Similarly, our existence is necessarily confined to a three-dimensional world, so the possibility of extra dimensions cannot be verified even if analogy permits understanding the nature of larger spaces. Ultimately, "objective reality" may be fundamentally unknowable, always a captive of our finite frames of reference. The consideration of scaling models is then just a case study in the limits of scientific methodology.

We may find it useful to adopt a spatial analogy in describing a political world, but the representation cannot convey all of that political world. We may be intrinsically limited to partial representations, confined by our conceptual bases, and seeking to describe a reality that can never be fully known. Partial images of an unknowable reality—a challenging agenda for a flight into Dimensionland.

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