Let $Z_{jk}$ be the $j$th stimulus coordinate on the $k$th dimension, $k=1,...,s$, where $s$ is the number of dimensions. Let $d_{jm}$ be the Euclidean distance between stimulus $j$ and stimulus $m$ in the $s$-dimensional space:

$$d_{jm} = \sqrt{\sum_{k=1}^{s}(Z_{jk} - Z_{mk})^2}$$  \hspace{1cm} (1)

We assume that the observed distances, $d_{jm}^*$, are

$$d_{jm}^* = d_{jm} + \epsilon_{jm}$$  \hspace{1cm} (2)

And that they are drawn from the log-normal distribution because distances are inherently positive:

$$\ln(d_{jm}^*) \sim N(\ln(d_{jm}), \sigma^2)$$  \hspace{1cm} (3)

That is

$$f(d_{jm}^*) = \frac{1}{(2\pi\sigma^2)^{1/2}d_{jm}^*} e^{-\frac{1}{2\sigma^2}((\ln(d_{jm}^*) - \ln(d_{jm}))^2)}$$  \hspace{1cm} (4)

The log-normal is a more realistic model of the noise process because, by definition of the log-normal, $d_{jm}^* > 0$ so that $d_{jm} + \epsilon_{jm} > 0$ and $\sigma > 0$, where $\sigma$ is a shape parameter. The mean and variance are:
\[ E[d^*_{jm}] = d_{jm} e^{\frac{1}{2\sigma^2}} \quad \text{and} \quad \text{VAR}(d^*_{jm}) = (e^{\sigma^2} - 1)d_{jm}^2 e^{\sigma^2} \quad (5) \]

So that as \( d_{jm} \to 0 \), \( E(d^*_{jm}) \to 0 \) and \( \text{VAR}(d^*_{jm}) \to 0 \). The upshot is that the smaller the observed distance the smaller the variance of that distance because \( d^*_{jm} = d_{jm} + \epsilon_{jm} \).

Our likelihood function is:

\[
\mathbf{L}^*(Z_{jk}|D^*) = \frac{1}{(2\pi\sigma^2)^{q(q-1)/2}} \prod_{j=1}^{q-1} \prod_{m=j+1}^{q} \frac{1}{d^*_{jm}} e^{-\frac{1}{2\sigma^2} \sum_{j=1}^{q-1} \sum_{m=j+1}^{q} \left[ \ln(d^*_{jm}) - \ln\left( \sqrt{\sum_{k=1}^{q}(Z_{jk} - Z_{mk})^2} \right) \right]^2} \quad (6)
\]

To implement a Bayesian model we use simple normal prior distributions for the stimuli coordinates:

\[
\xi(Z_{jk}) = \frac{1}{(2\pi\kappa^2)^{1/2}} e^{-\frac{Z_{jk}^2}{2\kappa^2}} \quad (7)
\]

and a uniform prior for the variance term:

\[
\xi(\sigma^2) = \frac{1}{c}, \quad 0 < c < b \quad (8)
\]

Where, empirically, \( b \) is no greater than 2.

Hence, our posterior distribution is:

\[
\xi(Z_{jk}|D^*) \propto \prod_{j=1}^{q-1} \prod_{m=j+1}^{q} \left\{ f_{jm}(Z_{jm}|d^*_{jm})\xi(Z_{11})\xi(Z_{12})...\xi(Z_{is})\xi(Z_{21})...\xi(Z_{qs})\xi(\sigma^2) \right\} \quad (9)
\]
Taking the log of the right hand side and dropping the unnecessary constants:

\[
\ell n_{\xi} \propto -\frac{q(q-1)/2}{2} \ln(\sigma^2) - \frac{1}{2\sigma^2} \sum_{j=1}^{q-1} \sum_{m=j+1}^{q} \left( \ln\left( d_{jm}^* \right) - \ln\left( \sqrt{\sum_{k=1}^{s} (Z_{jk} - Z_{mk})^2} \right) \right)^2 \\
- \frac{1}{2\kappa^2} \left( \sum_{j=1}^{q} \sum_{k=1}^{s} Z_{jk}^2 \right) - \ln(c) = \\
-\frac{q(q-1)/2}{2} \ln(\sigma^2) - \frac{1}{2\sigma^2} \sum_{j=1}^{q-1} \sum_{m=j+1}^{q} \left( \ln\left( d_{jm}^* \right) - \ln\left( d_{jm} \right) \right)^2 - \frac{1}{2\kappa^2} \left( \sum_{j=1}^{q} \sum_{k=1}^{s} Z_{jk}^2 \right) - \ln(c) \tag{10}
\]